

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION (S), MAY 2019

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks.

Marks

- 1 a) Check the convergence of the series $\sum_{k=1}^{\infty} \left(\frac{3k-4}{4k-5}\right)^k$ (2)
- b) Find the Maclaurin series of $f(x) = \frac{1}{1+x}$, up to 3 terms (3)
- 2 a) If $z = (3x - 2y)^4$, find $\frac{\partial^4 z}{\partial x \partial y^3}$ (2)
- b) If $w = \log(\tan x + \tan y + \tan z)$ then prove that (3)
 $\sin 2x \frac{\partial w}{\partial x} + \sin 2y \frac{\partial w}{\partial y} + \sin 2z \frac{\partial w}{\partial z} = 2$
- 3 a) Find the speed of a particle moving along the path $x = 2 \cos t, y = 2 \sin t, z = t$ (2)
at $t = \pi/2$
- b) If $y'(t) = \cos t \, i + \sin t \, j$; $y(0) = i - j$. Find $y(t)$. (3)
- 4 a) Evaluate $\int_0^1 \int_0^{x^2} \int_0^2 dy \, dz \, dx$ (2)
- b) Evaluate $\iint xy \, dx \, dy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and lying in the first quadrant. (3)
- 5 a) Show that $F(x, y) = 2xy^3 i + 3x^2 y^2 j$ is conservative. (2)
- b) If $\vec{r} = x \vec{i} + y \vec{j} + z \vec{k}$ and $r = \|\vec{r}\|$, prove that $\nabla \cdot \frac{\vec{r}}{r^3} = 0$ (3)
- 6 a) Evaluate by Stoke's theorem $\oint_C (e^x \, dx + 2y \, dy - dz)$, where C is the curve (2)
 $x^2 + y^2 = 4, z = 2$
- b) Using Green's theorem evaluate $\int_C x \, dy - y \, dx$ where C is the circle $x^2 + y^2 = 4$ (3)

PART B

Module 1

Answer any two questions, each carries 5 marks.

- 7 Test for convergence of the series $\sum_{k=1}^{\infty} \frac{1}{(8k^2 - 3k)^{1/2}}$ (5)
- 8 Find the radius of convergence and interval of convergence of the power series $\sum_{k=0}^{\infty} \frac{(2x-1)^k}{3^{2k}}$. (5)

- 9 Show that the series $\sum_{k=1}^{\infty} (-1)^k \left(\frac{k}{k+1}\right)^{k^2}$ is convergent. (5)

Module 1I

Answer any two questions, each carries 5 marks.

- 10 Let $w = \sqrt{x^2 + y^2 + z^2}$, where $x = \cos \theta$, $y = \sin \theta$, $z = \tan \theta$. Find $\frac{dw}{d\theta}$ at $\theta = \frac{\pi}{4}$, using chain rule. (5)
- 11 Find the local linear approximation $L(x, y)$ to $f(x, y) = \ln(xy)$ at the point $P(1, 2)$. Compare the error in approximating f by L at the point $Q(1.01, 2.01)$ with the distance between P and Q . (5)
- 12 Find relative extrema and saddle points, if any, of the function (5)
- $$f(x, y) = xy + \frac{8}{x} + \frac{8}{y}.$$

Module 1II

Answer any two questions, each carries 5 marks.

- 13 Find the unit tangent $T(t)$ and unit normal $N(t)$ to the curve (5)
- $$x = a \cos t, y = a \sin t, z = ct \quad a > 0$$
- 14 Find the velocity and position vectors of the particle, if the acceleration vector (5)
- $$a(t) = \sin t \mathbf{i} + \cos t \mathbf{j} + e^t \mathbf{k}; \quad v(0) = \mathbf{k}; \quad r(0) = -\mathbf{i} + \mathbf{k}.$$
- 15 Find the equation of the tangent line to the curve of intersection of surfaces (5)
- $$z = x^2 + y^2 \text{ and } 3x^2 + 2y^2 + z^2 = 9 \text{ and the point } (1, 1, 2).$$

Module 1V

Answer any two questions, each carries 5 marks.

- 16 Evaluate by reversing the order of integration $\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2 - y^2}} x \, dx \, dy$ (5)
- 17 Evaluate $\iint_R xy \, dA$, where R is the sector in the first quadrant bounded by (5)
- $$y = \sqrt{x}, y = 6 - x, y = 0.$$
- 18 Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz \, dx \, dy$ (5)

Module V

Answer any three questions, each carries 5 marks.

- 19 Find the work done by $F(x, y) = (x^2 + y^2)i - xj$ along the curve $C: x^2 + y^2 = 1$ counter clockwise from $(1, 0)$ to $(0, 1)$ (5)
- 20 Determine whether $F(x, y) = 6y^2 i + 12xy j$ is a conservative vector field. If so find the potential function for it. (5)
- 21 Find the divergence and curl of the vector field $F(x, y, z) = xyz^2 i + yzx^2 j + zxy^2 k$ (5)
- 22 Prove that $\int_C (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k \cdot d\vec{r}$ is independent of the path and evaluate the integral along any curve from $(0, 0, 0)$ to $(1, 2, 3)$. (5)
- 23 If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = \|\vec{r}\|$, prove that $\nabla^2 f(r) = \frac{2}{r} f'(r) + f''(r)$. (5)

Module VI

Answer any three questions, each carries 5 marks.

- 24 Using Green's theorem evaluate $\int_C (xy + y^2)dx + x^2 dy$ where C is the boundary of the region bounded by $y = x^2$ and $x = y^2$ (5)
- 25 Evaluate the surface integral $\iint_{\sigma} z^2 ds$, where σ is the portion of the curve $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 3$ (5)
- 26 Determine whether the vector field $F(x, y, z)$ is free of sources and sinks. If not, locate them. (5)
- i) $F(x, y, z) = (y + z)i - xzj + x^2 \sin y k$
 ii) $F(x, y, z) = x^3 i + y^3 j + 2z^3 k$
- 27 Use divergence theorem to find the outward flux of the vector field $F(x, y, z) = (2x + y^2)i + xy j + (xy - 2z)k$ across the surface σ of the tetrahedron bounded by $x + y + z = 2$ and the coordinate planes. (5)
- 28 Using Stoke's theorem evaluate $\int_C \vec{F} \cdot d\vec{r}$; where $\vec{F} = xy\vec{i} + yz\vec{j} + xz\vec{k}$; C triangular path in the plane $x + y + z = 1$ with vertices at $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ in the first octant (5)
